

# Welcome to Day 2!!!!

- Please complete the Day 1 Reflection Form and place on the back table before we begin the session today.
- Pick up one of every handout from the side table and place behind tab 3 in your binder.

# Reflection

What implications may the experience of the Task Sort have on my teaching?

What and how can I share important pieces of the Task Sort back on my campus?

Please answer these questions on an index card and place in the back sleeve cover of your binder for “safe keeping”



# Algebraic Reasoning Content Academy – Grade 5

Day 2: Multiple Representations  
Sami Briceño

# Adding It Up: Helping Children Learn Mathematics

- Conceptual Understanding: comprehension of concepts, operations and relations
- Procedural Fluency: skill in flexibly, accurately, efficiently, and appropriately using procedures
- Strategic Competence: ability to formulate, represent and solve problems
- Adaptive Reasoning: capacity for logical thought, reflection, explanation, and justification
- Productive Disposition: view of math as useful, sensible and worthwhile coupled with belief in diligence and personal efficacy



Kilpatrick, J., Swafford, J., Findell, B. (Ed.). (2001). Adding it up: helping children learn mathematics. Washington, DC: National Academy Press.

# Problem Solving—Van De Walle

- The problem must begin where the students are
- The problematic or engaging aspect of the problem must be due to the mathematics
- The problem must require justifications and explanations for answers and methods.

# Show & Tell Approach—Van De Walle

Listen to a description from Van de Walle's book of outcomes in a teacher directed approach.

# Problem Solving—Van De Walle

- Focuses students' attention on ideas and sense making.
- Develops the belief in students that they are capable of doing mathematics and that mathematics makes sense.
- Provides ongoing assessment data.
- Excellent method for attending to a breadth of abilities.
- Develops “mathematical power.”
- IT IS A LOT OF FUN

# Provide Hints but Not Solutions —Van De Walle

“Always keep in mind that as soon as students sense that you have a method of solving the problems, they will almost certainly stop searching for their own methods because they are convinced that your way must be best.”

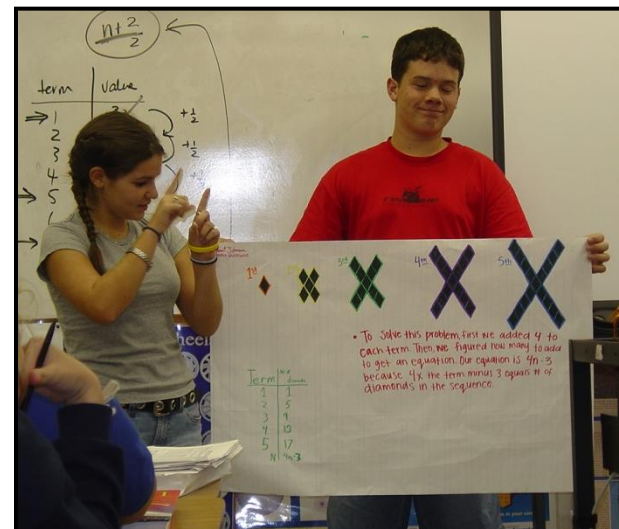


# Gr 3-5 Teaching Student-Centered Mathematics—Van De Walle pg 21-24

- What can I tell them? Should I tell them anything?
- How will I be able to teach all of the basic skills?
- Why is it okay for a student to “tell” or “explain” but not for me?
- This approach take more time, how will I have time to cover everything?
- Do I need to use a problem-based approach every day?
- Is there any place for drill and practice?
- My textbook is traditional, how can I use it?
- What do I do when a task bombs or students don’t “get it”?

Require students to defend and talk about their solutions because communication deepens understanding.

**When did the mathematics make sense for you?**



#1 Answer:

When I started teaching it!!!

# Day 2 Learning Outcomes

- Use both real-life and abstract representations to solve problems
- Investigate what can be learned from analyzing students' work
- Introduce the value of multiple representations: table, verbal/pictorial, equation, graph
- Translate and make connections among multiple representations, including technology
- Solve, analyze, and discuss 5th grade algebraic reasoning tasks, connection to MS & HS Algebra Tasks

# Bird Watching



- One of Justin's hobbies is bird watching. He was excited because he had moved to a new home in an area that had a large bird population. On the first day he saw 2 birds near his home and on the second day he saw 5 birds near his home. On the third day he saw 8 birds, and on the fourth day he saw 11 birds. If this pattern continued, how many birds would Justin see on the tenth day? How many would he see on the hundredth day?
- Extension: Write a rule for finding how many birds he will see on any number of days.

# Analyzing Student Work

- Examine each students' response. What does it suggest the student understands? Does not understand? Why?
  - Which response do you think shows the greatest understanding? Why?
  - Which response do you think shows the least understanding? Why?
- Rank your group's student responses from weakest to strongest using a 1, 2, 3, and 4 ranking. 1 being weakest understanding.

# Analyzing Student Work

- What are the characteristics of a response that indicates understanding?
- What are the characteristics of a response that indicates a lack of understanding?

# Sample A

I need to find how many b/hds will I just in 100 days.  
day. I will make a table

day	birds
1	2
2	5
3	8
4	11
5	14
6	17
7	20
8	23
9	26
10	29
11	32

+ 11 + 11 + 11 = 33 - 1 = 32

1 x 3 = 26

on day 100  
 $100 + 100 + 100 = 300 - 1 = 299$



# Sample B

You have to find how many birds

Birds he saw

days	birds
1	2
2	5
3	8
4	11

2b he saw 2b birds on day 10  
he saw 12b birds on day 100

# Sample C

I need to find how many birds Justin saw,  
 I know that he saw 2 birds on the first  
 day and 3 more  
 than that.

I noticed that they  
 are counting  
 by 3's

Birdwatching


days	birds
1	2
2	5
3	8
4	11
5	14
6	17
7	20
8	23
9	26
10	29

$3 \times d - 1 = b$

key  
 d is day  
 b is bird

$100 \times 3 - 1 = 299$

$9 \times 3 - 1 = 26$   
 $3 \times 9 - 1 = 26$



# Sample D

I have to tell you the birds he  
saw on day 10 and 100.

1-2  
2-5  
3-8 birds  
day 4-11 birds  
5-14  
6-16  
7-19  
8-22  
9-25  
10-28 ←

100 day is 128

# Analyzing Student Work

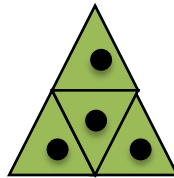
- What are the characteristics of a response that indicates understanding?
- What are the characteristics of a response that indicates a lack of understanding?

# Betty's Miniature Bird House Business

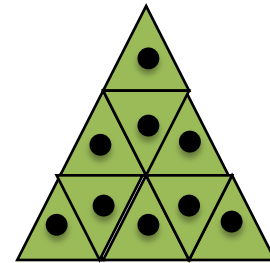
Betty has a bird house business. She sells miniature bird houses through a catalog. Here is a page from one of her catalogs:



Catalog Item #1:  
A Single Home  
(1 bird can live here)



Catalog Item #2:  
Condominium  
(4 birds can live here)



Catalog Item #3:  
High Rise Apartment

If this pattern of miniature bird houses continues, determine the number of miniature birds that could live in miniature bird houses listed as catalog items #10, #100, and #500.

# Variation on Bird House Business-- Growing Triangles



Figure 1

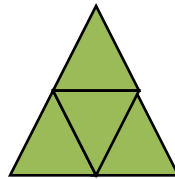


Figure 2

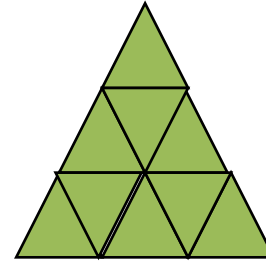


Figure 3

- Using pattern blocks, construct the 3 figures above.
- Describe the pattern you notice in the figures.

# Growing Triangles



Figure 1

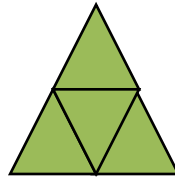


Figure 2

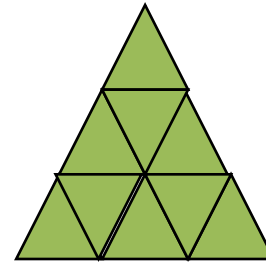


Figure 3

- How many triangles are needed to create the next figure?
- How many triangles are needed to create the 10<sup>th</sup> figure? 15<sup>th</sup> figure? 25<sup>th</sup> figure?
- Write a rule representing the relationship between the figure number, ***f***, and the number of triangles needed to make the figure, ***n***.

# Growing Triangles

- Create your own growing pattern using any pattern blocks of the SAME shape. Draw the first three figures.
- What is the algebraic rule for your growing pattern between the figure number and the number of triangles needed to construct the figure?



# Tile Patterns



# Tiling around a Fountain

- **SCENARIO:** You are working on a landscaping crew for the county parks department this summer.
- To avoid the mud that surrounded the park fountains last summer, your crew is planning to put a border of tiles around each of the square fountains in the park. The border tiles each measure 1-foot on each side. Your foreman shows you this diagram for the smallest fountain. You notice that a fountain that has a base of 1 square foot will require 8 border tiles.
- Using this pattern, how many tiles will be needed for different size square fountains?



# Tiling around a Fountain

**Problem 1:** If a square fountain has sides of length  $s$  feet, how many tiles are needed to form the border?

- Using grid paper draw a diagram of the designs for the border of fountains with side lengths of 2, 3, 4, 6 and 10 feet. Record your results in a table.
- Write an equation for the number of tiles,  $N$ , needed to form a border for a square fountain with side length of  $S$  feet.
- Generate as many equations as you can for this relationship.
  - Are the equations the same?
  - How can you convince someone that your expressions for the number of tiles needed are equivalent?



# Group Poster

## Tiling around a Fountain

- Show all the expressions generated by your group
- Draw Fountain with Side Length 3, explicitly connect all of your expressions to this diagram



# Gallery Walk

## Tiling around a Fountain

- One person from each group “mans” the group’s poster to answer questions.
- Rest of the group members view other posters. Look for:
  - The most common representations
  - The most unique representations





# Re-Tiling

Tiling around a Fountain

**Follow-up Problem:** Make a table and a graph for each equation you found in problem 1.

- Do the tables and graphs show that the expressions and equations are equivalent?
- Is this relationship between  $s$  and  $n$  a straight line, or a curve?



# Collection of Expressions

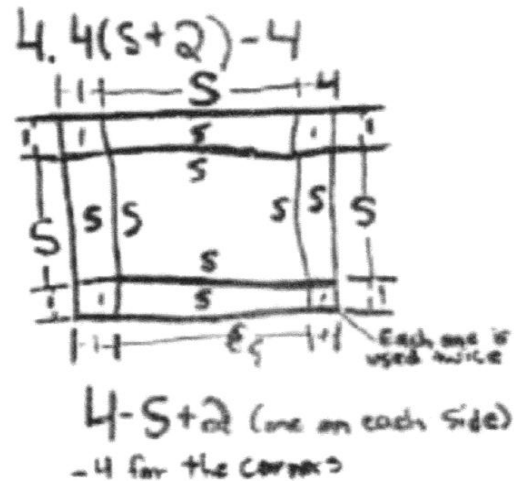
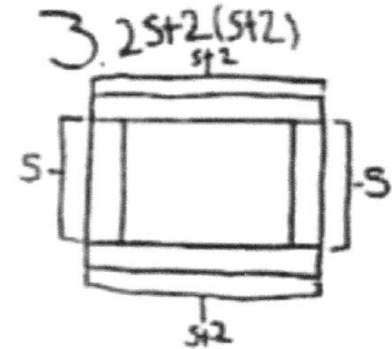
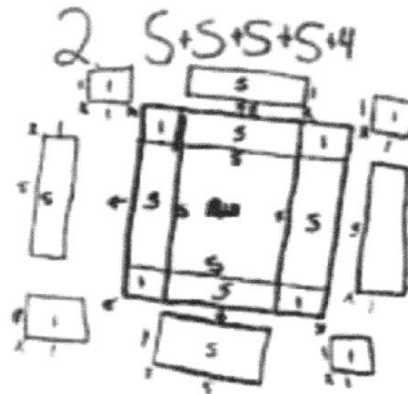
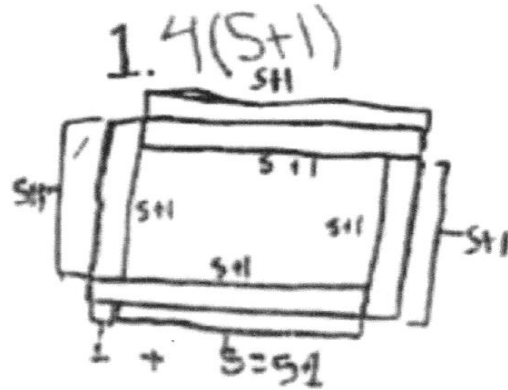
Tiling around a Fountain

- Make a collective list of all the expressions that were generated in the groups.

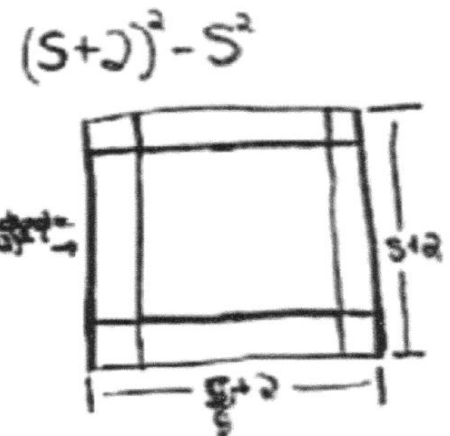
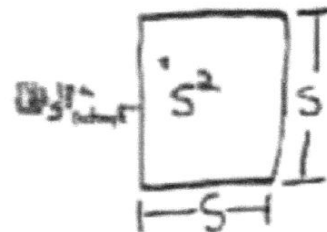


# One Group's Work

## Tiling around a Fountain



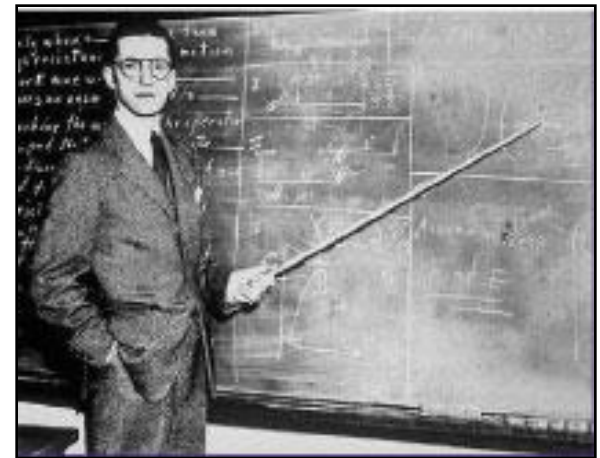
5. 4 corners = 4  
 4 sides =  $s+s+s+s$





# Looking through Teacher Lenses

- How would you characterize the level of this task: High or low cognitive demand?
- What mathematical ideas are embedded in the task?
- What makes this worthwhile mathematics?



# Why do we want kids to experience Pattern Tasks?

- Understanding variables
- Connecting between representations
- Progressing on standards
- Using context for working on functions
- Starting the problem at multiple entry points
- Finding and comparing multiple solutions



# Telephone Task

Scenario: A cellular telephone company charges \$10.00 per month plus \$0.20 per call. Complete the table and graph the results.

Number of Calls	0	1	2	3	4	5	10	20
Total Cost (Dollars)								

Write an algebraic rule to determine the total cost,  **$c$** , when given the number of calls,  **$n$** .

# Model Car Racing Task

The distance formula  $d = 0.25t$  represents the distance,  $d$ , a model racing car travels in  $t$  minutes at a rate of 0.25 kilometers per minute.

Labels	Time	Distance
	Minutes	Kilometers
Unit		

# Model Car Racing Task

Labels	Time	Distance for Model Car #1	Distance for Model Car #2
	Minutes	Kilometers	Kilometers
Unit	$t$	$0.25t$	$0.5t$

- As the time traveled increases, what happens to the distance traveled?
- How do the values for the distance traveled of Model Car #2 compare to the distance traveled by Model Car #1?
- How is the graph of the 2<sup>nd</sup> car different from the graph of the 1<sup>st</sup> car?

# U.S. Shirts

Using Tables, Graphs, and Equations

This past summer you were hired to work at a custom T-shirt shop, U.S. Shirts. One of your responsibilities is to find the total cost of customers' orders. The shop charges \$8 per shirt with a one-time set-up fee of \$15.

1. What is the total cost of an order for 10 shirts?
2. What is the total cost of an order for 100 shirts?
3. Explain how you found the total costs.
4. How many T-shirts can a customer buy for \$60?
5. How many T-shirts can a customer buy for \$250?
6. Explain how you found the number of shirts that can be purchased.



# U.S. Shirts

Using Tables, Graphs, and Equations

Make a table of values for the problem situation.

Labels	Number of Shirts Ordered	Total cost
	Shirts	\$
Unit		



# U.S. Shirts

Using Tables, Graphs, and Equations

- Create a graph of the data from your table. First, choose your bounds and intervals. Remember to label your graph clearly and add a title to the graph.
- Use your graph to determine the price of 40 shirts and 27 shirts. Use your graph to determine how many shirts can be purchased for \$300 and for \$540.





# U.S. Shirts

Using Tables, Graphs, and Equations

- Write an algebraic equation for the problem situation.
- In this lesson, you have represented the problem situation in four different ways: as a sentence, as a table, as a graph, and as an equation. Explain the advantages and disadvantages of each representation.



# Looking through Teacher Lenses

- How would you characterize the level of this task: High or low cognitive demand?
- What mathematical ideas are embedded in the task?
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